Philipp Legner, Mathigon

The Virtual Classroom

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Fluency and Understanding: A Mathematically Balanced World
Thursday 10.45 (CBA0.060) • Thursday 11.55 (CBA0.13) • Friday 9.00 (CB0.060)

Hi! I'm Philipp, and during the last few years I created a mathematics outreach website called Mathigon. I started working on Mathigon at University and have added to it ever since. The objective of Mathigon is to show how exciting and colourful mathematics is – using a variety of animations, videos, eBooks, games and more. Throughout the talk I'll show you various small parts of Mathigon as examples.

Unlike many of you I'm not a teacher, but I have taught in many different schools as part of outreach or research projects. And one thing I noticed was that at all stages, from primary to university, the process of learning looked something like this:

- First, students are told an algorithm, a theorem or a process how to solve a particular kind of problem.
- Then students memorise this algorithm.
- Finally students apply the algorithm to solve many similar-ish problems. And usually these *problems* are in fact *exam questions* which students are being trained for.

Of course this diagram is incredibly simplified and generalised. Reality is certainly more interesting, in particular, when students are given a more complex problem and have to work out which results to use and how to apply them. Still – there are few aspects that seem to be missing and which I think are incredibly important.

- Lessons often start with a fact or theorem and then move on to problems and applications. But this means that usually there is very little *motivation* or *reason* why to study a particular topic. I think a more natural process would be to start with an application or puzzle, then arrive at a problem and come up with new ways to solve it.

  For example, this textbook chapter on game theory starts with a simple interactive game involving boxes of chocolates. Students will quickly discover that the computer always wins the game, and the remainder of the article is about how to develop winning strategies for this and other games.

  This chapter starts with a puzzle regarding the existence of triangles with three right angles on the surface of the sphere. This unexpected result then motivates a more detailed exploration of the properties of spherical and hyperbolic geometry.

- Very much related to this, I think that lessons or textbook shouldn't just be a collection of facts, explanations and problems. There should be a narrative that wraps the content and puts it into context. This makes the content more interesting and exciting, but also more relevant to students and easier to remember.

  For example, this textbook chapter on circles and conic sections tells a chronological or biographic story, from the ancient Greeks to Kepler, Newton and the scientific revolution.

  This eBook tells stories based on applications of mathematical ideas: like Poisson processes that make phones and the internet work, or Calculus that helps Roller Coaster Design, or exponential decay that underlies carbon dating.

  The story could even be fictional: here is an interactive video that follows "Alice in Fractalland", as she discovers sequences, Pascal's Triangle, Fibonacci Numbers and fractals.

- Most importantly, though, it seems that students are always *told* how to solve certain problems – they never discover new solutions on their own. For me, this is the most exciting part of mathematics: exploring, playing, finding patterns, generalising, and then coming up with a solution or algorithm.

  Rather than telling students Pythagoras' Theorem, and then telling them to apply it in lots of different problems, we could let students play with triangles, maybe guide and nudge them occasionally, and let them discover this amazing theorem on their own.

  Or we could ask very open ended question – like *"How many ways are there to arrange these 5 chairs?"* – and let them come up with a solution entirely on their own.

  Education would immediately be much more about creativity than memorising, where the teacher acts more like a "Guide" than an "Instructor". Of course you could argue that this approach might be too slow or too difficult for some students, but particularly mathematics education is just as much about learning problem solving, as it is about learning useful facts and knowledge.

- Finally, it seems that all students follow exactly the same curriculum, no matter what their skills and abilities are, or what they are interested in. Ideally, every student would have their own, personalised curriculum, and – to an extend – students might even be able to decide on their own what they are interested in and what they want to learn.

Unfortunately, all of these ideas are impossible in classrooms where a single teacher has to take care of 20 or 30 students, all with different interests and abilities, under a lot of time pressure, and where the only valid success criterium is examination results.

But the title of this talk was "The *Virtual* Classroom" – so in the second half of this talk I want to explore how some of these ideas could be achieved in the future using the power of computers, the internet and artificial intelligence.

During the last couple of years there has been an explosion of education technology projects, companies and websites.

Khan Academy contains hundreds of videos on a wide range of subjects, as well as many problems. The website tracks how well you solve problems and which videos you watch, in order to provide even better content the next time.

Massive Open Online Courses like **Udacity**, **Udemy**, **Coursera** or **EdX** make highest quality educational content much more widely accessible – to thousands if not millions of students.

And there are entirely new technologies like **Geogebra** for geometry and **Desmos** for graphs, which allow students to explore and play with mathematics like never before.

On all these platforms, however, the content itself – and the way it is taught – is still very much the same as before – and indeed, not much different from education 100 years ago. So, just imagining, what would we want education of the future to look like?

- We want students to be *motivated* to learn – this is particularly important for online education, where often you don't have due dates, exams, encouragement from teachers or peer support.

  Motivation can come by giving students a reason to study a topic, and by using storytelling to put the content in context and to make it more relevant and exciting. We could also use Gamification, visual media and animations.

All together, learning should be empowering: I once heard someone say *"textbooks should make students feel like superheroes"*, learning skills and abilities they didn't have before. This is often used in educational games, where you can reach higher levels, earn badges or beat your friends [competitiveness, pride of accomplishment]. And I think we need more of that in all other kinds of education.

- We want students to be able to explore and discover solutions, stimulating creativity rather than memorising skills.
- We want the curriculum to be streamlined to individual students, moving at an appropriate speed, and we even want the content, explanations and hints to be personalised to particular student's skills and needs.

I've been thinking quite a lot about these ideas, and how they could be achieved in an online platform. And to conclude I want to show you the first prototype for a completely new kind of online textbook: Mathigon Active.

Mathigon Active Demo (mathigon.org/active)

Little initial content • Solve blanks • Reveal more content • Progress sidebar • Personalise upcoming content • Textbook Mode • Illustrations • Interactive Variables • Biographies and Glossary • Videos • Play Slider • Help Sidebar • Arrow Pointers • Königsberg bridges (wrong and correct answer) • Step-by-step slideshows • Four colour maps • Audio narration • Discussions sidebar • Bookmarks and pins

Mathigon Active is still under development and this is just the very first chapter, which was mainly designed to showcase all the different features. The idea was to simulate, as much as possible, a human teacher sitting next to you and guiding you step-by-step through a new problem. Rather than separating content and exercises, the exercises become part of the content, as students "discover" more sections.

Mathigon Active will observe your interactions and learning style and track your knowledge and ability, and then try to adapt and personalise upcoming content – although not all of that functionality is finished yet!

Over time, I hope to create an entire curriculum based on these textbooks. From the list of chapters you can see that while there are Arithmetic and Equations and Trigonometry, the focus is on discovering exciting mathematical results – rather than all the facts and algorithms in the current mathematics curriculum, which many students will never actually use after leaving school.

Time for Questions

Thanks for coming. Hope you found the talk interesting and please email me if you have any comments and suggestions, or just want to talk!

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